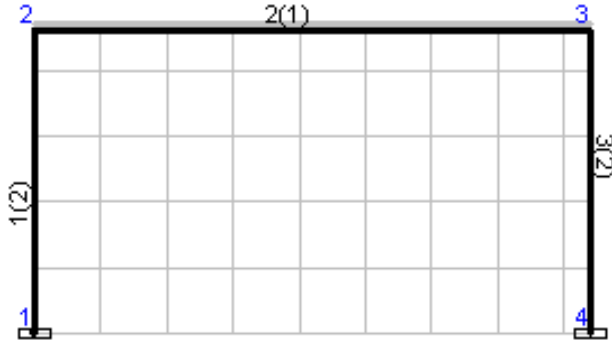


1-Finite element model (FEM)



Nodal points

Node	x [m]	y [m]
1	0.000	0.000
2	0.000	4.600
3	8.400	4.600
4	8.400	0.000

Supports

Node	kind	ux [mm]	uy [mm]	ur [rad]
1	fixed	ux=uy=ur=0		
4	fixed	ux=uy=ur=0		

Materials

Material : R.Concrete, E= 26.000 [GPa]
 Weight density : $\rho = 25.000$ [kN/m³]
 The element self weight is included in loads and masses

Element cross sections

Cr.sec.	b [cm]	h [cm]	b1 [cm]	h1 [cm]	Ac [cm ²]	Ic [cm ⁴]
1	25.000	70.000	120.000	15.000	3.17500E+003	1.33529E+006
2	30.000	60.000			1.80000E+003	5.40000E+005

Elements

Element	node 1	node 2	material	length(m)	angle(°)
1	1	2	2	4.600	90.000
2	2	3	1	8.400	0.000
3	3	4	2	4.600	270.000

Element distributed loads ($\gamma_g=1.35$, $\gamma_q=1.50$)

element	G [kN/m]	Q [kN/m]	$\gamma_g G + \gamma_q Q$ [kN/m]	load kind	load direction
2	22.500	12.400	48.975	uniform	vertical

Element distributed loads due to self weight ($\gamma_g=1.35$, $\gamma_q=1.50$)

element	G [kN/m]	Q [kN/m]	$\gamma_g G + \gamma_q Q$ [kN/m]	load kind	load direction
1	4.500	0.000	6.075	uniform	vertical
2	7.937	0.000	10.715	uniform	vertical
3	4.500	0.000	6.075	uniform	vertical

2-Results of static-linear-elastic analysis

Diagrams of internal forces M, V, N, and displacements d, of element 1

n	x/l	x [m]	M [kNm]	V [kN]	N [kN]	dx [mm]	dy [mm]	d [mm]
0	0.000	0.00	103.67	67.91	-278.64	0.000	0.000	0.000
1	0.100	0.46	72.43	67.91	-275.85	-0.070	-0.026	0.075
2	0.200	0.92	41.19	67.91	-273.05	-0.250	-0.052	0.255
3	0.300	1.38	9.95	67.91	-270.26	-0.491	-0.078	0.497
4	0.400	1.84	-21.29	67.91	-267.46	-0.748	-0.104	0.755
5	0.500	2.30	-52.53	67.91	-264.67	-0.972	-0.130	0.981
6	0.600	2.76	-83.77	67.91	-261.88	-1.118	-0.156	1.128
7	0.700	3.22	-115.01	67.91	-259.08	-1.137	-0.182	1.151
8	0.800	3.68	-146.25	67.91	-256.29	-0.982	-0.208	1.004
9	0.900	4.14	-177.49	67.91	-253.49	-0.608	-0.234	0.651
10	1.000	4.60	-208.73	67.91	-250.70	0.035	-0.260	0.263

Maximum values for element 1

maxM= 103.67 kNm, minM= -208.73 kNm
 maxV= 67.91 kN, minV= 67.91 kN
 maxN= -250.70 kN, minN= -278.64 kN
 maxd= 1.151 mm

Diagrams of internal forces M, V, N, and displacements d, of element 2

n	x/l	x [m]	M [kNm]	V [kN]	N [kN]	dx [mm]	dy [mm]	d [mm]
0	0.000	0.00	-208.73	-250.70	-67.91	0.035	-0.260	0.263
1	0.100	0.84	-19.20	-200.56	-67.91	0.028	-1.850	1.850
2	0.200	1.68	128.21	-150.42	-67.91	0.021	-3.486	3.486
3	0.300	2.52	233.51	-100.28	-67.91	0.014	-4.869	4.869
4	0.400	3.36	296.68	-50.14	-67.91	0.007	-5.784	5.784
5	0.500	4.20	317.74	0.00	-67.91	0.000	-6.103	6.103
6	0.600	5.04	296.68	50.14	-67.91	-0.007	-5.784	5.784
7	0.700	5.88	233.51	100.28	-67.91	-0.014	-4.869	4.869
8	0.800	6.72	128.21	150.42	-67.91	-0.021	-3.486	3.486
9	0.900	7.56	-19.20	200.56	-67.91	-0.028	-1.850	1.850
10	1.000	8.40	-208.73	250.70	-67.91	-0.034	-0.260	0.263

Maximum values for element 2

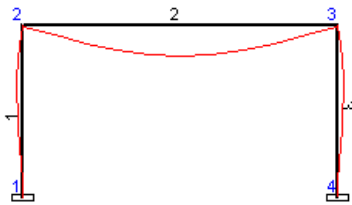
maxM= 317.74 kNm, minM= -208.73 kNm
 maxV= 250.70 kN, minV= -250.70 kN
 maxN= -67.91 kN, minN= -67.91 kN
 maxd= 6.103 mm

Diagrams of internal forces M, V, N, and displacements d, of element 3

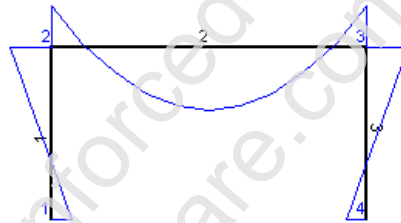
n	x/l	x [m]	M [kNm]	V [kN]	N [kN]	dx [mm]	dy [mm]	d [mm]
0	0.000	0.00	-208.73	-67.91	-250.70	-0.034	-0.260	0.263
1	0.100	0.46	-177.49	-67.91	-253.49	0.607	-0.234	0.651
2	0.200	0.92	-146.25	-67.91	-256.29	0.982	-0.208	1.004
3	0.300	1.38	-115.01	-67.91	-259.08	1.137	-0.182	1.151
4	0.400	1.84	-83.77	-67.91	-261.88	1.118	-0.156	1.129
5	0.500	2.30	-52.53	-67.91	-264.67	0.972	-0.130	0.980
6	0.600	2.76	-21.29	-67.91	-267.46	0.748	-0.104	0.755
7	0.700	3.22	9.95	-67.91	-270.26	0.491	-0.078	0.497
8	0.800	3.68	41.19	-67.91	-273.05	0.250	-0.052	0.255
9	0.900	4.14	72.43	-67.91	-275.85	0.071	-0.026	0.075
10	1.000	4.60	103.67	-67.91	-278.64	0.000	0.000	0.000

Maximum values for element 3

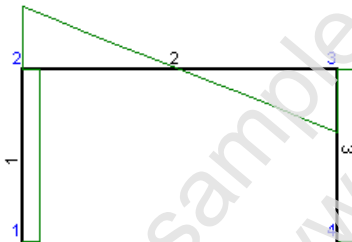
maxM= 103.67 kNm, minM= -208.73 kNm
 maxV= -67.91 kN, minV= -67.91 kN
 maxN= -250.70 kN, minN= -278.64 kN
 maxd= 1.151 mm



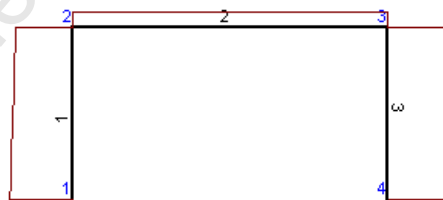
Displacement diagram
maxD=6.10 mm



Bending moment diagram
maxM=317.74 kNm, minM=-208.73 kNm



Shear force diagram
maxV=250.70 kN, minV=-250.70 kN



Axial force diagram
maxN=-67.91 kN, minN=-278.64 kN

3-Dimensioning of Concrete

Design codes

EN1990:2002, Eurocode 0 Basis of Structural Design
 EN1991-1-1:2002, Eurocode 1-1 Actions on structures
 EN1992-1-1:2004, Eurocode 2 Reinforced concrete
 EN1997-1-1:2004, Eurocode 7 Geotechnical design
 EN1998-1-1:2004, Eurocode 8 Design in earthquake environment
 NA - National Annex:

Concrete-Steel class: C25/30-B500C (EC2 §3)
 Concrete cover : C_{nom}=30 mm (EC2 §4.4.1)
 $\gamma_c=1.50$, $\gamma_s=1.15$ (EC2 Table 2.1N)
 $f_{cd}=\alpha_{cc} \cdot f_{ck} / \gamma_c = 0.85 \times 25 / 1.50 = 14.17$ MPa (EC2 §3.1.6)
 $f_{yd} = f_{yk} / \gamma_s = 500 / 1.15 = 435$ MPa (EC2 §3.2.7)
 Concrete weight : 25.0 kN/m³

Reinforced concrete design, element 1, L= 4.600m, B= 300mm, H= 600mm

Med= 184.96 kNm, Ved= 67.91 kN, Ned= -278.64 kN
 L=4.600m. L_{cz}=8.650m =1.88xL, L_{cy}=8.650m=1.88xL

Dimensions and loads

Column of rectangular cross section b=0.300 m, h=0.600 m, column length L=4.600 m
 Loads, axial Ned=278.64 kN (compression), moments Med_{yy}=184.96 kNm, Med_{zz}=0.00 kNm
 shear Ved=67.91 kN
 Effective length direction z-z : L_{cz}= 1.88xL= 8.650m
 Effective length direction y-y : L_{cy}= 1.88xL= 8.650m
 Effective depth of cross section d=h-d₁, d₁=d₂=C_{nom}+Ø_s+Ø/2=30+8+20/2=48mm, dx=252mm, dy=552mm

Design for compression with small eccentricity (UL3)

(EC2 §6.1, §9.2.1)

Ned=278.64kN, Med_{yy}=184.96kNm, Med_{zz}=0.00kNm

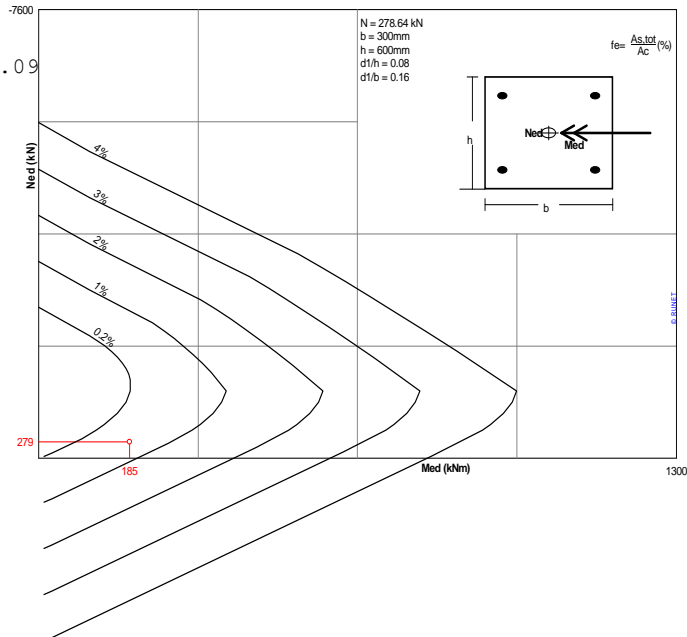
Approximate design from Tables (d₁/h=0.10)

Kordina K, Bemessungshilfsmittel zu EC 2 Teil 1
 Planung von Stahlbeton ..., Berlin, Beuth, 1992
 $M_y / (b h^2 f_{cd}) = 0.10$, $M_z / (b h^2 f_{cd}) = 0.00$, $N / (b h f_{cd}) = -0.09$
 $A_s \cdot f_{yk} / (b h \cdot f_{ck}) = 0.18$, $A_s = 1242 \text{ mm}^2$, $A_s / A_c = 0.69\%$

Design using numerical integration

Design chart for single bending and axial force
 obtained from numerical integration of the
 concrete and steel forces over the cross-section
 Ned=278.64kN (compression), Med=184.96kNm
 C25/30-B500C
 b=300mm, h=600mm
 d=552mm, d₁= 48mm, d₂= 48mm, d₁/h=0.080
 $e = \text{Med} / \text{Ned} = 184.96 / 278.64 = 0.664 \text{ m} = 664 \text{ mm}$
 $z_s = h / 2 - d_1 = 600 / 2 - 48 = 252 \text{ mm}$, $e = 664 \text{ mm} > z_s = 252 \text{ mm}$
 $A_{s1} = A_{s2} = 675 \text{ mm}^2$, $(A_{s1} + A_{s2}) / A_c = 0.75\%$
 $\epsilon_{c2} / \epsilon_{s1} = -3.50 / -10.50$

As₁= 675mm², As₂= 675mm²
 As=1350mm²



Minimum longitudinal reinforcement, $A_s \geq 0.0020A_c$, $\phi_s \geq 8$, $A_{s,min} = 4\phi 12$ (452mm²) (EC2 §9.5.2.2)
 Maximum longitudinal reinforcement, $A_s \leq 0.04A_c$, ($A_{s,max} = 7200\text{mm}^2$) (EC2 §9.5.2.3)
 Transverse reinforcement, links with minimum ϕ_s at maximum spacing $S_{cl,t}$ (EC2 §9.5.3)
 at column heights from 0.60m to H-0.60m: Links $\phi_s \geq 6$, $S_{cl,t} \leq 300\text{mm}$
 at regions 0 to 0.60m and H-0.60m to H: Links $\phi_s \geq 6$, $S_{cl,t} \leq 180\text{mm}$
 Basic required anchorage length $L_{bd} = 570\text{mm} = 0.570m$ (EC2 Eq.8.3)

Longitudinal reinforcement: 8Ø20 (2512mm²)

Transverse reinforcement: Links Ø 8/300 [h:0.60m~H-0.60m], **Ø 8/180** [h:0~0.60m, H-0.60m~H]

Ultimate limit state (ULS), Design for shear (EC2 EN1992-1-1:2004, §6.2, §9.2.2)

Shear capacity without shear reinforcement V_{rdc} (EC2 §6.2.2)
 $V_{rdc} = [C_{rdc} \cdot k \cdot (100\rho_l \cdot f_{ck})^{0.33} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d$ (EC2 Eq.6.2.a)
 $V_{rdc} \geq (v_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d$ (EC2 Eq.6.2.b)
 $C_{rdc} = 0.18 / \gamma_c = 0.18 / 1.50 = 0.120$, $f_{ck} = 25.00\text{MPa}$, $b_w = 300\text{mm}$, $d = 552\text{mm}$
 $k = 1 + \sqrt{(200/d)} \leq 2$, $k = 1.60$, $k_1 = 0.15$
 $\rho_l = A_{s1} / (b_w \cdot d) = 1250 / (300 \times 552) = 0.0075$
 $\sigma_{cp} = N_{ed} / A_c = 1000 \times 278.64 / 180000 = 1.55\text{N/mm}^2$
 $v_{min} = 0.035 \cdot k^{1.50} \cdot \sqrt{f_{ck}} = 0.35\text{N/mm}^2$ (EC2 Eq.6.3N)
 $V_{rd,c(min)} = 0.001 \times (0.35 + 0.15 \times 1.55) \times 300 \times 552 = 96.46\text{kN}$
 $V_{rdc} = 0.001 \times [0.120 \times 1.60 \times (0.75 \times 25.00)^{0.33} + 0.15 \times 1.55] \times 300 \times 552 = 122.97\text{kN}$
 $V_{ed} = 67.91\text{kN} \leq V_{rdc} = 122.97\text{kN}$, **Ved < Vrdc shear reinforcement is not needed**

Concrete strut capacity V_{rdmax} (EC2 §6.2.3 Eq.6.9)
 $V_{rdmax} = \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd} / (\cot\theta + \tan\theta)$, $V_{ed} / \max(V_{rdmax}) = 0.12$, $\theta = 45.0^\circ$ $\cot\theta = 1.00$ $\tan\theta = 1.00$
 $\alpha_{cw} = 1.00$ $z = 0.9d$, $f_{ck} = 25.0 \leq 60\text{Mpa}$ $v_1 = 0.6 [1 - f_{ck}/250] = 0.6 [1 - 25/250] = 0.540$, $f_{cd} = 14.17\text{Mpa}$
 $V_{rdmax} = 0.001 \times 1.00 \times 300 \times 0.9 \times 552 \times 0.540 \times 14.17 / 2.00 = 570.2\text{kN}$

Design for second order effects (EC2 EN1992-1-1:2004, §5.8.3)

Final creep coefficient $\phi(\infty, t_0) = 2.50$ (EC2 §3.1.4, Annex B)
 Effective creep coefficient $\phi_{ef} = \phi(\infty, t_0) \cdot (M_{oEq} / M_{oEd}) = 2.50 \times 0.50 = 1.25$ (EC2 §5.8.4)
 Modulus of elasticity of concrete $E_{cd} = E_{cm} / \gamma_{ce} = 1000 \times 31 / 1.20 = 25.42\text{GPa} = 25417\text{MPa}$ (EC2 Eq.5.20)
 Modulus of elasticity of steel $E_s = 200\text{GPa} = 200000\text{MPa}$
 Reinforcement ratio $\rho = A_s / (b \cdot d) = 2512 / (300 \times 600) = 0.014$

Slenderness criterion for isolated members (EC2 EN1992-1-1:2004, §5.8.3.1)

$\lambda_{lim} = 20 \cdot A \cdot B \cdot C / \sqrt{n}$ (Eq.5.13N)
 $\omega = A_s \cdot f_{yd} / (A_c \cdot f_{cd}) = 2512 \times 435 / (300 \times 600 \times 14.17) = 0.43$
 $n = N_{ed} / (A_c \cdot f_{cd}) = 278643 / (300 \times 600 \times 14.17) = 0.109$
 $A = 1 / (1 + 0.2 \cdot \phi_{ef}) = 1 / (1 + 0.2 \times 1.25) = 0.80$
 $B = \sqrt{1 + 2.0 \cdot \omega} = \sqrt{1 + 2.0 \cdot 0.43} = 1.36$
 $C = 1.70 - r_m = 0.70$, ($r_m = M_{01} / M_{02} = 1.0$)
 $\lambda_{lim} = 20 \times 0.80 \times 1.36 \times 0.70 / \sqrt{0.109} = 46.17$

Slenderness and effective length, direction z-z (EC2 EN1992-1-1:2004, §5.8.3.2)

Unbraced members $\beta = L_0 / L = \max(\sqrt{[1 + 10 \cdot k_1 \cdot k_2 / (k_1 + k_2)]}, [1 + k_1 / (1 + k_1)] [1 + k_2 / (1 + k_2)])$ (Eq.5.16)
 Effective length $L_0 = \beta \cdot L = 1.88 \times 4.600 = 8.650\text{m}$
 Slenderness ratio $\lambda = L_0 / i$, $i = 0.289 \times 600\text{mm}$, $\lambda = 8650 / 173 = 49.88$ (Eq.5.14)
 $\lambda = 49.88 > \lambda_{lim} = 46.17$, **second order effects must be considered**

Slenderness and effective length, direction y-y (EC2 EN1992-1-1:2004, §5.8.3.2)

Unbraced members $\beta = L_0 / L = \max(\sqrt{[1 + 10 \cdot k_1 \cdot k_2 / (k_1 + k_2)]}, [1 + k_1 / (1 + k_1)] [1 + k_2 / (1 + k_2)])$ (Eq.5.16)
 Effective length $L_0 = \beta \cdot L = 1.88 \times 4.600 = 8.650\text{m}$
 Slenderness ratio $\lambda = L_0 / i$, $i = 0.289 \times 300\text{mm}$, $\lambda = 8650 / 87 = 99.77$ (Eq.5.14)
 $\lambda = 99.77 > \lambda_{lim} = 46.17$, **second order effects must be considered**

Nominal Stiffness

(EC2 EN1992-1-1:2004, §5.8.7.2)

$EI = K_c \cdot E_{cd} \cdot I_c + K_s \cdot E_s \cdot I_s$ (EC2 Eq.5.21)

$\rho = A_s / A_c = 0.014$, $E_{cd} = 25417 \text{ MPa}$, $E_s = 200000 \text{ MPa}$
 $n = N_{ed} / (A_c \cdot f_{cd}) = 278643 / (300 \times 600 \times 14.17) = 0.109$

$K_s = 1$, $K_c = k_1 \cdot k_2 / (1 + \phi_{ef})$, $\phi_{ef} = 1.25$ (EC2 Eq.5.22)

$k_1 = \sqrt{(f_{ck} / 20) \text{ MPa}} = \sqrt{(25 / 20)} = 1.12 \text{ MPa}$ (EC2 Eq.5.23)

direction z-z

$k_2 = n \cdot \lambda / 170 \leq 0.20$, $n = 0.109$, $\lambda = 49.88$, $k_2 = 0.032$ (EC2 Eq.5.24)

$K_c = 1.118 \times 0.032 / (1 + 1.25) = 0.016$

$EI = 0.016 \times 25417 \times 300 \times 600^3 / 12 + 1.0 \times 200000 \times 1256 \times (552 / 2)^2 = 21.32 \cdot 10^{12} \text{ Nmm}^2 = 21322 \text{ kNm}^2$

direction y-y

$k_2 = n \cdot \lambda / 170 \leq 0.20$, $n = 0.109$, $\lambda = 99.77$, $k_2 = 0.064$

$K_c = 1.118 \times 0.064 / (1 + 1.25) = 0.032$

$EI = 0.032 \times 25417 \times 600 \times 300^3 / 12 + 1.0 \times 200000 \times 1256 \times (252 / 2)^2 = 5.08 \cdot 10^{12} \text{ Nmm}^2 = 5081 \text{ kNm}^2$

Moment magnification factor

(EC2 EN1992-1-1:2004, §5.8.7.3)

$M_{ed} = M_{oed} [1 + \beta / ((N_b / N_{ed}) - 1)]$, $N_b = \pi^2 \cdot EI / L_o^2$ (EC2 Eq.5.28)

direction z-z

$\beta = \pi^2 / c_o$, $c_o = 12.0$, $\beta = 0.82$ (EC2 Eq.5.29)

$N_b = 3.14^2 \times 21322 / 8.650^2 = 2812.48 \text{ kN}$

$M_{ed} / M_{oed} = 1 + 0.82 / (2812.48 / 278.64 - 1) = 1.09$, **Med, yy = 201.68 kNm**

direction y-y

$\beta = \pi^2 / c_o$, $c_o = 12.0$, $\beta = 0.82$ (EC2 Eq.5.29)

$N_b = 3.14^2 \times 5081 / 8.650^2 = 670.25 \text{ kN}$

$M_{ed} / M_{oed} = 1 + 0.82 / (670.25 / 278.64 - 1) = 1.59$, **Med, zz = 0.00 kNm**

Design for compression with small eccentricity (ULS)
 (with second order effects)

(EC2 §6.1, §9.2.1)

Ned = 278.64 kN, Med, yy = 201.68 kNm, Med, zz = 0.00 kNm

Approximate design from Tables ($d_1/h = 0.10$)

Kordina K, Bemessungshilfsmittel zu EC 2 Teil 1 Planung von Stahlbeton ..., Berlin, Beuth, 1992

$M_y / (b h^2 f_{cd}) = 0.11$, $M_z / (b h^2 f_{cd}) = 0.00$, $N / (b h \cdot f_{cd}) = -0.09$

$A_s \cdot f_{yk} / (b h \cdot f_{ck}) = 0.20$, $A_s = 1380 \text{ mm}^2$, $A_s / A_c = 0.77\%$

Design using numerical integration

Design chart for single bending and axial force obtained from numerical integration of the concrete and steel forces over the cross-section

Ned = 278.64 kN (compression), Med = 201.68 kNm

C25/30-B500C

b = 300 mm, h = 600 mm

d = 552 mm, d1 = 48 mm, d2 = 48 mm, d1/h = 0.080

e = Med / Ned = 201.68 / 278.64 = 0.724 m = 724 mm

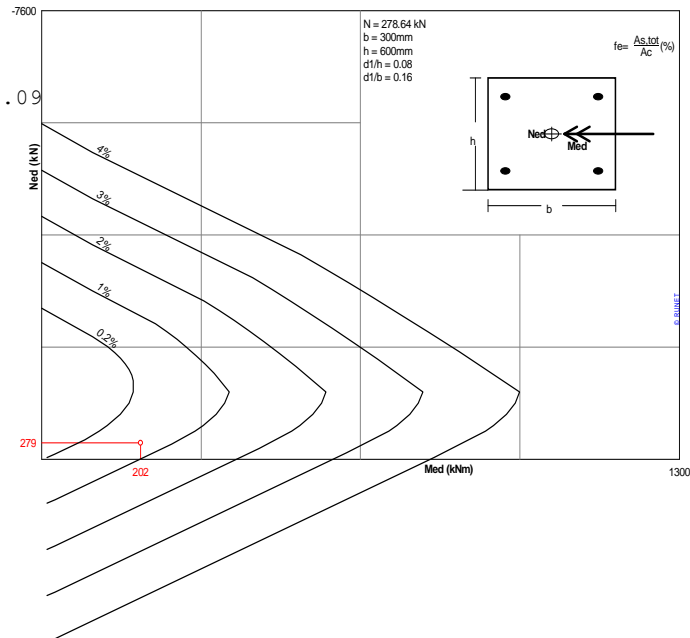
zs = h / 2 - d1 = 600 / 2 - 48 = 252 mm, e = 724 mm > zs = 252 mm

As1 = As2 = 675 mm², (As1 + As2) / Ac = 0.75%

εc2 / εs1 = -3.50 / -10.50

As1 = 675 mm², As2 = 675 mm²

As, tot = 1350 mm²



Minimum longitudinal reinforcement, $A_s \geq 0.0020A_c$, $\phi_s \geq 8$, $A_{s,min} = 4\phi 12$ (452mm²) (EC2 §9.5.2.2)
 Maximum longitudinal reinforcement, $A_s \leq 0.04A_c$, ($A_{s,max} = 7200\text{mm}^2$) (EC2 §9.5.2.3)
 Transverse reinforcement, links with minimum ϕ_s at maximum spacing $S_{cl,t}$ (EC2 §9.5.3)
 at column heights from 0.60m to H-0.60m: Links $\phi_s \geq 6$, $S_{cl,t} \leq 300\text{mm}$
 at regions 0 to 0.60m and H-0.60m to H: Links $\phi_s \geq 6$, $S_{cl,t} \leq 180\text{mm}$
 Basic required anchorage length $L_{bd} = 570\text{mm} = 0.570m$ (EC2 Eq.8.3)

Longitudinal reinforcement: 8Ø20 (2512mm²)

Transverse reinforcement: Links Ø 8/300 [h:0.60m~H-0.60m], **Ø 8/180** [h:0~0.60m, H-0.60m~H]

Reinforced concrete design, element 2, [Span], L= 8.400m, B= 250mm, H= 700mm

Med = 317.74 kNm, Ved = 0.00 kN, Ned = -67.91 kN (x=4.20m)

Dimensions and loads

Beam web width $b_w = 0.250$ m, beam height $h = 0.700$ m
 Effective flange width $b_{eff} = 1.200$ m, slab thickness $h_f = 0.150$ m
 Effective depth of cross section $d_1 = c_{nom} + \phi_s + 1.1\phi = 30 + 8 + 1.1 \times 16 = 56\text{mm}$, $d_2 = 56\text{mm}$, $d = 700 - 56 = 644\text{mm}$

Ultimate limit state (ULS)

Bending moment $M_{ed} = 317.74$ kNm, Shear $V_{ed} = 0.00$ kN, Axial force $N_{ed} = -67.91$ kN (compression)

Serviceability limit state (SLS)

Bending moment $M_{ed} = 158.87$ kNm, Shear $V_{ed} = 0.00$ kN, Axial force $N_{ed} = -33.96$ kN (compression)

Ultimate limit state (ULS), design for bending with axial force

(EC2 §6.1, §9.2.1)

Reinforcement for bending with axial force (only tension reinforcement is needed)

Dimensioning for bending: Allgower, G.-Avak, R. Bemessungstabeln nach Eurocode 2 für Rechteck und Plattenbalkenquerschnitte, In: Beton - und Stahlbetonbau 87 (1992)

$M_{ed} = 318\text{kNm}$, $N_{sd} = -68\text{kN}$, $b_{eff} = 1200\text{mm}$, $d = 644\text{mm}$, $K_d = 1.215$, $x/d = 0.08$, $\epsilon_c / \epsilon_{s1} = -1.7/20.0$, $k_s = 2368$, **$A_{s1} = 1085\text{mm}^2$**

$x = 0.08 \times 644 = 52 < h_f = 150\text{mm}$ neutral axis within the depth of top flange

Minimum longitudinal tension reinf., $A_s \geq 0.26bd \cdot f_{ctm} / f_{yk}$, ($A_{s,min} = 218\text{mm}^2$) (EC2 §9.2.1.1.1)

Maximum tension or compression reinf., $A_s \leq 0.04A_c$, ($A_{s,max} = 7000\text{mm}^2$) (EC2 §9.2.1.1.3)

Longitudinal reinforcement: 6Ø16 (1206mm²) (bottom)

Ultimate moment capacity of cross section

(EC2 EN1992-1-1:2004, §6.1)

$b_w = 250\text{mm}$, $h = 700\text{mm}$, $b_f = 1200\text{mm}$, $h_f = 150\text{mm}$, $d = 644\text{mm}$, $A_{s1} = 1206\text{mm}^2$, $A_{s2} = 0\text{mm}^2$

$\epsilon_c = -1.70\text{‰}$, $\epsilon_{s1} = 19.91\text{‰}$, $A_{s1} / b \cdot d = 0.00156$ (0.156%)

$x/d = \epsilon_c / (\epsilon_c + \epsilon_{s1}) = 1.70 / (1.70 + 19.91) = 0.079$, $x = 50.7\text{mm}$

$\alpha_r = 0.609$, $\alpha_k = 0.366$, $F_c = \alpha_r \cdot b \cdot x \cdot f_{cd} = F_{s1} = 525.09\text{kN}$, $A_{s1} = F_{s1} / f_{yd} = 1207\text{mm}^2$

$z = d - \alpha_k \cdot x = (1 - \alpha_k \cdot \epsilon_c / (\epsilon_c + \epsilon_{s1})) d$, $z/d = 1.0 - 0.366 \times 0.079 = 0.971$, $z = 625.8\text{mm}$,

$K_d^2 = 1 / (0.609 \cdot 0.079 \cdot 0.971 \cdot 14.17) = 1.516$ mm²/N, $K_d = 1.231$

Bending capacity $M_r = b \cdot d^2 / K_d^2 = [10^{-6}] \times 1200 \times 644^2 / 1.516 = 329.00\text{kNm}$

$x = 50.7$ mm \leq $h_f = 150\text{mm}$ neutral axis in flange

Bending capacity ($\epsilon_c / \epsilon_{s1} = 1.70/19.91$) $M_r = 329.00\text{kNm}$

Shear between web and flanges

(EC2 EN1992-1-1:2004, §6.2.4)

$F_c = F_s = 0.001 \times 1206 \times 435 = 525 \text{ kN}$
 $\Delta F_d = F_c \cdot (b_{eff} - b_w) / (2b_{eff}) = 525.0 \times (1200 - 250) / (2 \times 1200) = 208 \text{ kN}$
 Beam span $L = 8.40 \text{ m}$, $\Delta x = 0.70 \times 8.40 / 4 = 1.47 \text{ m}$ (EC2 §5.3.2.1)
 $V_{rdmax} = v \cdot hf \cdot f_{cd} \cdot \sin \theta \cdot \cos \theta$, $f_{cd} = 14.17 \text{ MPa}$, $\theta = 26.5^\circ$
 $v = 0.6(1 - f_{ck}/250) = 0.54$ (EC2 Eq.6.6N)
 $V_{rdmax} = 0.54 \times 150 \times 14.17 \times \sin 26.5^\circ \times \cos 26.5^\circ = 458 \text{ kN/m}$ (EC2 Eq.6.22)
 $\Delta F_d / \Delta x = 208 / 1.47 = 141 < V_{rdmax} = 458 \text{ kN/m}$, the check is verified
 Transverse reinforcement per unit length A_{sf}/s_f (EC2 Eq.6.21)
 $A_{sf}/s_f = 1000 \times 141 / (435 \times \cot 26.5^\circ) = 162 \text{ mm}^2/\text{m}$

Transverse reinforcement $A_{sf}/s_f = \emptyset 8/310$ (162 mm²/m)

$\Delta F_d / \Delta x = 141 > 0.40 \cdot hf \cdot f_{ctd} = 0.40 \times 150 \times 1.02 = 61 \text{ kN/m}$

In case of transverse flexural reinforcement from plate bending, the steel area should be the greater or half the above, plus the required for transverse bending (EC2 §6.2.4.5)

Serviceability limit state (SLS)

(EC2 EN1992-1-1:2004, §7)

$M_{ed}(SLS) = 158.87 \text{ kNm}$, $N_{ed}(SLS) = -33.96 \text{ kN}$
 Final creep coefficient $\phi(\infty, t_0) = 2.50$ (EC2 §3.1.4, Annex B)
 Total shrinkage strain $\epsilon_{cs} = -0.30 \text{ o/o}$
 $\gamma_c = 1.00$, $\gamma_s = 1.00$ (EC2 §2.4.2.4.2)
 Modulus of elasticity of concrete $E_{cm} = 31 \text{ GPa}$, $E_{c,eff} = 31 / (1 + 2.50) = 8.71 \text{ GPa} = 8710 \text{ MPa}$ (EC2 Eq.7.20)
 Modulus of elasticity of steel $E_s = 200 \text{ GPa} = 200000 \text{ MPa}$
 Modular ratio $\alpha_e = E_s / E_c = 200 / 30.50 = 6.56$, effective $\alpha_e = E_s / E_{c,eff} = 200 / 8.71 = 22.96$
 Tension reinforcement: $\emptyset 16$
 Reinforcement ratio $\rho = A_s / (b \cdot d) = 1206 / (1200 \times 644) = 0.002$

State I (uncracked section) (SLS)

Bending stiffness of uncracked section, $EI = (200 / 22.96) \times (0.001 \times 32.700) = 284841 \text{ kNm}^2$
 $S = A_s \cdot z_{s1} = (0.001)^2 \times 1206 \times 0.171 = (0.001) \times 0.206 \text{ m}^3$ (EC2 Eq.7.21)
 Curvature due to moment $1/r_M = 158.870 / 284841 = (0.001) \times 0.558 \text{ (1/m)}$
 Curvature due to shrinkage $1/r_{cs} = (0.001 \times 0.30) \times 22.960 \times (0.206 / 32.700) = (0.001) \times 0.043 \text{ (1/m)}$
 Total curvature $1/r = (0.001) \times 0.558 + (0.001) \times 0.043 = (0.001) \times 0.601 \text{ (1/m)}$
 Cracking moment, $M_{cr} = f_{ctm} \cdot (I / y^2) = 2.6 \times (32.700 / 0.227) = 375.24 \text{ kNm}$

State II (fully cracked section) (SLS)

$\rho = A_s / (b \cdot d) = 0.002$, $n = \alpha_e = 22.96$, $n \cdot \rho = 0.046$, $\xi = 0.675$, $\alpha = 0.261$, $x = \alpha \cdot d = 0.168 \text{ m}$
 Bending stiffness of fully cracked section, $EI = \xi \cdot E_s \cdot A_s \cdot d^2 = 0.675 \times 200 \times 1206 \times 0.644^2 = 67625 \text{ kNm}^2$
 $S = A_s \cdot z_{s1} = (0.001)^2 \times 1206 \times 0.476 = (0.001) \times 0.575 \text{ m}^3$ (EC2 Eq.7.21)
 Curvature due to moment $1/r_M = 158.870 / 67625 = (0.001) \times 2.349 \text{ (1/m)}$
 Curvature due to shrinkage $1/r_{cs} = (0.001 \times 0.30) \times 22.960 \times (0.575 / 7.764) = (0.001) \times 0.121 \text{ (1/m)}$
 Total curvature $1/r = (0.001) \times 2.349 + (0.001) \times 0.121 = (0.001) \times 2.470 \text{ (1/m)}$
 $M_{ed} = 158.87 \text{ kNm}$, $N_{ed} = -33.96 \text{ kN}$, $\epsilon_c / \epsilon_s = 0.42 / 1.10$, $x = 177 \text{ mm}$, $\sigma_s = 219 \text{ N/mm}^2$

Checking deflections by calculation (SLS)

(EN1992-1-1, §7.4.3)

$M_{ed} = 158.87 < 0.70 \times M_{cr} = 0.70 \times 375.24 = 262.67 \text{ kNm}$, $\zeta = 0.00$ (Eq.7.19)
 Final curvature $(1/r) = 0.00 \times (0.001 \times 2.470) + (1 - 0.00) \times (0.001 \times 0.601) = (0.001) \times 0.601 \text{ (1/m)}$ (Eq.7.18)

Minimum reinforcement areas (SLS)

(EC2 EN1992-1-1:2004, §7.3.2)

Minimum reinforcement areas $A_{s,min} = k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct} / \sigma_s$ (EC2 Eq.7.1)
 $b = 0.250 \text{ m}$, $b_{eff} = 1.200 \text{ m}$, $h = 0.700 \text{ m}$, $d = 0.644 \text{ m}$, $x = 0.177 \text{ m}$, $\emptyset = 16 \text{ mm}$
 $N_{ed} = -33.96 \text{ kN}$, $\sigma_c = (N_{ed} / bh) = 0.2 \text{ N/mm}^2$, $\sigma_s = 219 \text{ N/mm}^2$
 $A_{ct} = (h - x) \cdot b = (700 - 177) \times 250 = 130648 \text{ mm}^2$
 $\max(h, b_1) = 1 \text{ mm}$, $f_{ctm} = 2.60 \text{ N/mm}^2$, $A_{c,eff} = 130648 \text{ mm}^2$, $k = 0.72$, $k_c = 0.35$, $k_1 = 0.67$
 Minimum reinforcement, $A_{s,min} = 0.35 \times 0.72 \times 2.60 \times 130648 / 219 = 390 \text{ mm}^2$

Calculation of crack width (SLS)

(EC2 EN1992-1-1:2004, §7.3.3)

$w_k = s_{r,max} \cdot (\epsilon_{sm} - \epsilon_{cm})$ (EC2 Eq.7.8)
 $\epsilon_{sm} - \epsilon_{cm} = [\sigma_s - k_t \cdot (f_{ct,eff} / \rho_{eff}) (1 + \alpha_e \cdot \rho_{eff})] / E_s \geq 0.6 \sigma_s / E_s$ (EC2 Eq.7.9)
 $\sigma_s = 219 \text{ N/mm}^2$, short term loading: $\alpha_e = 6.56$, $k_t = 0.6$, long term loading: $\alpha_e = 22.96$, $k_t = 0.4$
 $A_{ceff} = 2.5 (h - d) b = 2.5 \times (700 - 644) \times 250 = 34750 \text{ mm}^2$ (§7.3.2.3)
 $\rho_{eff} = A_s / A_{ceff} = 1206 / 34750 = 0.035$
 $\epsilon_{sm} - \epsilon_{cm} = [219 - 0.4 \times (2.6 / 0.035) (1 + 22.96 \times 0.035)] / 200 = 0.83 \text{ o/oo} \geq 0.6 \times 219 / 200 = 0.66 \text{ o/oo}$
 $s_{r,max} = k_3 \cdot (C_{nom} + \phi_s) + k_1 \cdot k_2 \cdot k_4 \cdot \phi / \rho_{eff}$ (EC2 Eq.7.11)
 $\phi = 16 \text{ mm}$, $k_1 = 0.8$, $k_2 = (e_1 + e_2) / 2e_1 = 0.5$, $k_3 = 3.4$, $k_4 = 0.425$
 $s_{r,max} = 3.4 \times 31.00 + 0.8 \times 0.5 \times 0.425 \times 16 / 0.035 = 183.77 \text{ mm}$
 $w_k = s_{r,max} \cdot (\epsilon_{sm} - \epsilon_{cm}) = 183.77 \times 0.001 \times 0.83 = 0.15 \text{ mm}$

Reinforced concrete design, element 2, [Left end], L= 8.400m, B= 250mm, H= 700mm

MedA=-136.20kNm (x=t/2=0.30m), VedA=198.77kN (x=t/2+d=0.97m), VedAmax=250.70kN, NedA=-67.91kN

Dimensions and loads

Beam web width $b_w = 0.250 \text{ m}$, beam height $h = 0.700 \text{ m}$
 Effective flange width $b_{eff} = 1.200 \text{ m}$, slab thickness $h_f = 0.150 \text{ m}$
 Effective depth of cross section $d_1 = C_{nom} + \phi_s + 0.5\phi = 56 + 8 + 0.5 \times 16 = 72 \text{ mm}$, $d_2 = 72 \text{ mm}$, $d = 700 - 72 = 628 \text{ mm}$

Ultimate limit state (ULS)

Bending moment $Med = -136.20 \text{ kNm}$, Shear $Ved = 198.77 \text{ kN}$, Axial force $Ned = -67.91 \text{ kN}$ (compression)

Ultimate limit state (ULS), design for bending with axial force

(EC2 §6.1, §9.2.1)

Dimensioning for bending: Allgower, G.-Avak, R. Bemessungstabellen nach Eurocode 2 für Rechteck und Plattenbalkenquerschnitte, In: Beton - und Stahlbetonbau 87 (1992)
 Reinforcement for bending with axial force (only tension reinforcement is needed)
 $Med = -136 \text{ kNm}$ $Nsd = -68 \text{ kN}$ $b_w = 250 \text{ mm}$ $d = 654 \text{ mm}$ $K_d = 0.826$ $x/d = 0.14$ $\epsilon_{c2} / \epsilon_{s1} = -3.2 / 20.0$ $k_s = 2437$, **$A_{s2} = 428 \text{ mm}^2$**
 Minimum longitudinal tension reinf., $A_s > 0.26 b d \cdot f_{ctm} / f_{yk}$, ($A_{s,min} = 221 \text{ mm}^2$) (EC2 §9.2.1.1.1)
 Maximum tension or compression reinf., $A_s \leq 0.04 A_c$, ($A_{s,max} = 7000 \text{ mm}^2$) (EC2 §9.2.1.1.3)

Longitudinal reinforcement: 4Ø16 (804mm²) (top)

Ultimate moment capacity of cross section

(EC2 EN1992-1-1:2004, §6.1)

$b = 250 \text{ mm}$, $h = 700 \text{ mm}$, $d = 654 \text{ mm}$, $A_{s1} = 804 \text{ mm}^2$, $A_{s2} = 0 \text{ mm}^2$
 $\epsilon_{c2} = -3.50 \text{ o/oo}$, $\epsilon_{s1} = 15.25 \text{ o/oo}$, $A_{s1} / b \cdot d = 0.00492 (0.492\%)$
 $x/d = \epsilon_{c2} / (\epsilon_{c2} + \epsilon_{s1}) = 3.50 / (3.50 + 15.25) = 0.187$, $x = 122.1 \text{ mm}$
 $\alpha_r = 0.810$, $k_a = 0.416$, $F_c = \alpha_r \cdot b \cdot x \cdot f_{cd} = F_{s1} = 350.09 \text{ kN}$, $A_{s1} = F_{s1} / f_{yd} = 805 \text{ mm}^2$
 $z = d - k_a \cdot x = ([1 - k_a \cdot \epsilon_{c2} / (\epsilon_{c2} + \epsilon_{s1})] d)$, $z/d = 1.0 - 0.416 \times 0.187 = 0.922$, $z = 603.2 \text{ mm}$,
 $K_d^2 = 1 / (0.810 \cdot 0.187 \cdot 0.922 \cdot 14.17) = 0.506 \text{ mm}^2 / \text{N}$, $K_d = 0.712$
 Bending capacity $M_r = b \cdot d^2 / K_d^2 = [10^{-6}] \times 250 \times 654^2 / 0.506 = 212.00 \text{ kNm}$

Ultimate limit state (ULS), Design for shear

(EC2 EN1992-1-1:2004, §6.2, §9.2.2)

Shear capacity without shear reinforcement V_{rdc} (EC2 §6.2.2)
 $V_{rdc} = [C_{rdc} \cdot k \cdot (100 \rho_1 \cdot f_{ck})^{0.33} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d$ (EC2 Eq.6.2.a)
 $V_{rdc} \geq (v_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d$ (EC2 Eq.6.2.b)
 $C_{rdc} = 0.18 / \gamma_c = 0.18 / 1.50 = 0.120$, $f_{ck} = 25.00 \text{ MPa}$, $b_w = 250 \text{ mm}$, $d = 654 \text{ mm}$
 $k = 1 + \sqrt{200/d} \leq 2$, $k = 1.55$, $k_1 = 0.15$
 $\rho_1 = A_{s1} / (b_w \cdot d) = 1206 / (250 \times 654) = 0.0074$
 $\sigma_{cp} = Ned / A_c = 1000 \times 67.91 / 317500 = 0.21 \text{ N/mm}^2$
 $v_{min} = 0.035 \cdot k^{1.50} \cdot \sqrt{f_{ck}} = 0.34 \text{ N/mm}^2$ (EC2 Eq.6.3N)
 $V_{rd,c(min)} = 0.001 \times (0.34 + 0.15 \times 0.21) \times 250 \times 654 = 60.74 \text{ kN}$
 $V_{rdc} = 0.001 \times [0.120 \times 1.55 \times (0.74 \times 25.00)^{0.33} + 0.15 \times 0.21] \times 250 \times 654 = 85.58 \text{ kN}$
 $Ved = 198.77 \text{ kN} > V_{rdc} = 85.58 \text{ kN}$, **Ved > V_{rdc} shear reinforcement is needed**

Concrete strut capacity Vrdmax

(EC2 §6.2.3 Eq.6.9)

$V_{rdmax} = \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd} / (\cot\theta + \tan\theta)$, $V_{ed} / \max(V_{rdmax}) = 0.45$, $\theta = 21.8^\circ$ $\cot\theta = 2.50$ $\tan\theta = 0.40$
 $\alpha_{cw} = 1.00$ $z = 0.9d$, $f_{ck} = 25.0 \leq 60 \text{ Mpa}$ $v_1 = 0.6[1 - f_{ck}/250] = 0.6[1 - 25/250] = 0.540$, $f_{cd} = 14.17 \text{ Mpa}$
 $V_{rdmax} = 0.001 \times 1.00 \times 250 \times 0.9 \times 654 \times 0.540 \times 14.17 / 2.90 = 388.3 \text{ kN}$
 $V_{ed} = 250.7 \text{ kN} < 388.3 \text{ kN} = V_{rdmax}$, the check is verified

Shear reinforcement of vertical linkss

(EC2 §6.2.3 Eq.6.8)

$V_{rds} = (A_{sw}/s) \cdot z \cdot f_{ywd} \cdot \cot\theta$, $V_{rds} = 198.77 \text{ kN}$, $z = 0.9d$, $f_{ywd} = 0.8 f_{yk} = 400.00 \text{ N/mm}^2$, $\cot\theta = 2.50$
 $A_{sw}/s = V_{rds} / (z \cdot f_{ywd} \cdot \cot\theta) = (1.0 \text{E}+006) \times 198.77 / (0.9 \times 654 \times 400 \times 2.50) = 338 \text{ mm}^2/\text{m}$ ($A_{sw}/s = 338 \text{ mm}^2/\text{m}$)
Required shear reinforcement: ($A_{sw}/s = 338 \text{ mm}^2/\text{m}$)

Minimum links for shear reinforcement

(EC2 §9.2.2)

Minimum shear reinforcement ratio $\rho_{w,min}$

(EC2 Eq.9.5N)

$\rho_{w,min} = (0.08 \times (f_{ck})^{0.5}) / f_{yk}$, $f_{ck} = 25 \text{ N/mm}^2$, $f_{yk} = 500 \text{ N/mm}^2$, $\rho_{w,min} = 0.0008$
 $\min A_{sw}/s = 1000 \times 0.0008 \times 250 \times \sin(90^\circ) = 200 \text{ mm}^2/\text{m}$

Maximum longitudinal spacing of links $s_{lmax} = 0.75d$ ($\leq 600 \text{ mm}$) = 490mm

(EC2 §9.2.2.6, Eq.9.6N)

Maximum transverse spacing of link legs $s_{tmax} = 0.75d$ ($\leq 600 \text{ mm}$) = 490mm

(EC2 §9.2.2.8, Eq.9.8N)

Minimum shear reinforcement links $\emptyset 8/490$ ($A_{sw}/s = 205 \text{ mm}^2/\text{m}$)

Shear reinforcement: links $\emptyset 8/295$ ($A_{sw}/s = 341 \text{ mm}^2/\text{m}$)